MAT-61256 Geometric Analysis

Partial Exam 1, **5.3.2014**

Examiner Sirkka-Liisa Eriksson

Solve four problems out of the problems 1–5 according to your choice. All the problems have the same value. Justify all your answers well. You may get partial credits even from problems that you have not been able to solve completely. No calculators or other material are aloud.

1. Define what is the Euclidean space $\mathbb{R}^n$ and verify that it is a normed space with respect to the norm
$$\|(x_1, ..., x_n)\| = \sqrt{x_1^2 + ... + x_n^2}.$$

2. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a continuous function that satisfies $\|u\| \leq f(u)$ for all $u \in \mathbb{R}^n$. Verify that a set $f^{-1}([a, b])$ is compact for all $a, b \in \mathbb{R}$ and $a < b$.

3. Let $U \subset \mathbb{R}^n$ and $f : U \to \mathbb{R}^m$ is differentiable.
   (a) What does it mean that a function f is differentiable?
   (b) Verify that if a function f is differentiable then $p_i(f'(x)(h)) = \sum_{j=1}^{n} h_j D_j f_i(x)$.

4. Let
$$f(x, y_1, y_2) = x^2 y_1 + e^x + y_2.$$
Does there exists a differentiable function g in some neighborhood of $(1, -1)$ such that $g(-1, 1) = 0$ and $f(g(y_1, y_2), y_1, y_2) = 0$. If yes compute the derivative of this mapping at $(1, -1)$.

5. Explain what is $Cl_{0,3}$?