

You may freely choose four problems. Justify your solutions carefully, also the calculations. No calculators.

1. What is the image of the point  $(-1, 1)$  when it is first reflected across the line passing through origin and whose direction vector is  $(-1, 2)$ , and then across the line passing through the origin and whose direction vector is  $(1, 1)$ ? What is the image of  $(-1, 1)$  when the order of the preceding reflections are changed?
2. Explain the concepts
  - (a) the Clifford algebra  $Cl_{p,q}$ ?
  - (b) the exterior algebra?
  - (c) what is their relations?
3. Prove that the even subalgebra  $Cl_{3,0}^+$  of  $Cl_{3,0}$  is isomorphic with the algebra of quaternions.
4. Compute using quaternions in  $\mathbb{R}^3$  the formula of the rotation around the axes  $u = -i + 2j + k$  with the angle  $\frac{\pi}{2}$ . Then compute the formula when the preceding result is rotated around the axis  $v = -i + j + k$  with the angle  $\frac{\pi}{3}$ . What is the formula if you do the rotations in the different order.
5. If  $\{v_1, \dots, v_{n-1}\}$  is a linearly independent set in  $\mathbb{R}^n$ . How do you find a vector perpendicular (with respect to the usual scalar product) to the vector space spanned by  $\{v_1, \dots, v_{n-1}\}$  using Clifford algebras? Verify your claim.