4. Johda interpoloivan kuutiosplinin yhtälöt

$$h_{i+1}g_{i-1}+2(h_i+h_{i+1})g_i+h_ig_{i+1} = \frac{3h_{i+1}(f_i - f_{i-1})}{h_i}+\frac{3h_i(f_{i+1} - f_i)}{h_{i+1}} \quad (i \in \{1, \dots, n-1\})$$

Vihje: Hermiten kuutiopolynomi, joka interpoloi $f_i = f(x_i)$ ja $g_i = f'(x_i)$, noudattaa välillä $x_{i-1} \le x \le x_i$ kaavaa

$$s(x) = (1 + 2u_i)(1 - u_i)^2 f_{i-1} + (3 - 2u_i)u_i^2 f_i + u_i(1 - u_i)^2 h_i g_{i-1} + (u_i - 1)u_i^2 h_i g_i,$$

jossa $h_i = x_i - x_{i-1}$ ja $u_i = (x - x_{i-1})/h_i$.

Derive the interpolating cubic spline's equations

$$h_{i+1}g_{i-1}+2(h_i+h_{i+1})g_i+h_ig_{i+1} = \frac{3h_{i+1}(f_i - f_{i-1})}{h_i}+\frac{3h_i(f_{i+1} - f_i)}{h_{i+1}} \quad (i \in \{1, \dots, n-1\})$$

Hint: the Hermite cubic polynomial that interpolates $f_i = f(x_i)$ and $g_i = f'(x_i)$ is defined on the interval $x_{i-1} \le x \le x_i$ by the formula

$$s(x) = (1 + 2u_i)(1 - u_i)^2 f_{i-1} + (3 - 2u_i)u_i^2 f_i + u_i(1 - u_i)^2 h_i g_{i-1} + (u_i - 1)u_i^2 h_i g_i,$$

where $h_i = x_i - x_{i-1}$ and $u_i = (x - x_{i-1})/h_i$.