FYS-6606 Photonics

Exam 3.4.2014

Evaluation of the course:

Exam 50 % , Home assignments 50 %

Programmable calculator is allowed (laptop is forbidden)

Sheet of equations is allowed (provided with the exam)

Select **5 out of 6** exercises (or 5 with the lowest points will be counted)

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1. Define/explain the terms **briefly** (max. 5 sentences): Dispersion relation, effective index, chirped fiber Bragg grating, phase matching, group velocity, shot noise. (6p)

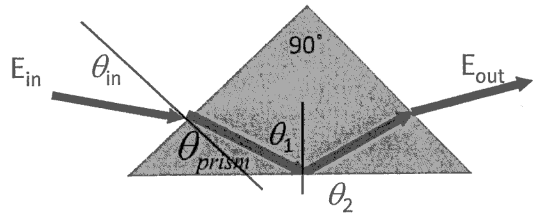
2. Cavity of a tunable Fabry-Perot interferometer is made of slab of silicon and two mirrors on its surfaces. Refractive index of silicon near 1550 nm is: $n_{eff}(\lambda,T)=n_0+n'(\lambda_0-\lambda)+\alpha(T-T_0)$, n_0=3.47771, n'=8.3e-5 nm^{-1}, λ_0=1550 nm, T_0=300K, α=1.5e-4 K^{-1}. Use T=300K if not othervise instructed.

a) Desing a filter for optical telecommunications with its peak **exactly at 1550 nm** and linewidth less than 1 GHz. What should the length of the cavity and reflectivity of the mirrors be? *Note: Maximum length of the cavity is determined by the thickness of 160 um of the slab. (2p)*

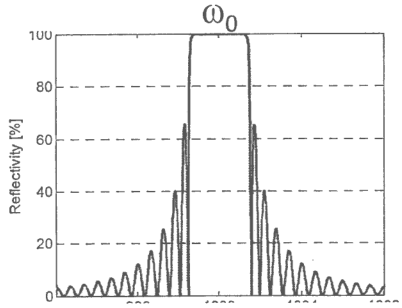
b) What is the FSR of the designed filter? (2p)

c) How much the temperature needs to be tuned to shift the peak position 100 GHz? (2p)

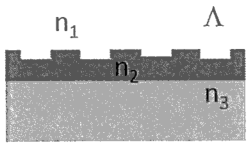
3. The figure illustrates **total internal reflection** (n_{prism}=1.45, n_{air}=1.0, λ=633 nm).
a) What is the angle θ_{in} when total internal reflection at the bottom surface takes place? (3p)
b) What is the ratio of output intensity to input intensity at this angle for TE-polarization? (3p)



4. **Fiber Bragg grating** has a sinusoidally varying refractive index.
   a) Explain the applications of this grating. (2p)
   b) Calculate the expression for reflectivity at the center of the reflection band from the coupled mode equations (at ω_0). (2p)
   c) How long grating is needed to achieve a reflectivity of 99.9% (κ=1000 1/m)? (2p)



5. Asymmetric slab waveguide.
Substrate (n_s=1.7), waveguide (n_{wg} =2.0) and thickness d=800 nm, cladding is air.
a) Explain the principle of a grating coupler to excite the modes of the waveguide from air (3p).
b) Design a grating coupler (period of the grating) to excite the TE_1- mode at 800 nm ($n_{eff}(TE_1)$=1.8375). Give the period of the grating and incident angle.
(Note: many solutions are possible)



6. The attenuation of a 1550 nm wavelength in an InGaAsP material is α = 6.0 cm^{-1}. Reflectivities of the laser cavity mirrors of a Fabry-Perot laser are R_1 = 0.9 and R_2 is the reflection from the air-semiconductor interface, respectively. Effective refractive index of the waveguide mode of the laser is n_{eff} = 3.4 and it's group index is 3.7.

a) Determine the gain threshold, G_{th}, necessary to achieve laser action if the cavity length L=500 µm. (2p)

b) What are the lasing wavelengths near 1550 nm? What is the spacing $\Delta\lambda$ between these modes? (2p)

c) How could you modify the structure to achieve single-mode operation (only a single wavelength is lasing)? (2p)

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Maxwell's equations and boundary conditions

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B} \qquad \mathbf{u}_n \times \mathbf{E}_1 = \mathbf{u}_n \times \mathbf{E}_2$$
$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega \mathbf{D} \qquad \mathbf{u}_n \times \mathbf{H}_1 = \mathbf{u}_n \times \mathbf{H}_2$$
$$\nabla \cdot \mathbf{D} = \rho \qquad \mathbf{u}_n \cdot \mathbf{D}_1 = \mathbf{u}_n \cdot \mathbf{D}_2$$
$$\nabla \cdot \mathbf{B} = 0 \qquad \mathbf{u}_n \cdot \mathbf{B}_1 = \mathbf{u}_n \cdot \mathbf{B}_2$$

Wave-equation and wavevector

$$\nabla^2 \mathbf{E} + \omega^2 \varepsilon \mu \mathbf{E} = 0 \quad |\mathbf{k}|^2 = \omega^2 \varepsilon \mu = \frac{\omega^2}{c_0^2} n^2$$

Planewaves

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{r}}$$
$$\mathbf{H}_0 = \frac{1}{\omega\mu} \mathbf{k} \times \mathbf{E}_0$$

Intensity of the planewave [W/m²]

$$I = \frac{k}{2\omega\mu}|E|^2 = \frac{c_0\varepsilon_0}{2}\sqrt{\varepsilon_r}|E|^2 = \frac{c_0\varepsilon_0}{2} n |E|^2$$

Complex refractive index

$$n = n_0 + j\kappa \qquad E_1 = E_0 e^{-jn_0 \frac{\omega}{c_0} z} e^{\kappa \frac{\omega}{c_0} z} \qquad \alpha = 2\kappa \frac{\omega}{c_0} = \kappa \frac{4\pi}{\lambda_0}$$

$\kappa<0$, Loss
$\kappa>0$, Gain

Transfer matrix method

Reflection / Transmission coefficients of planewaves can be calculated from the wavevectors k_1 and k_2 in the two media.

$$k_1^2 = \frac{\omega^2}{c_0^2} n_1^2 \qquad k_2^2 = \frac{\omega^2}{c_0^2} n_2^2$$

k_t tangential component
k_n normal component

At boundary $k_{t1} = k_{t2}$

$$k_{n1}^2 = k_1^2 - k_{t1}^2 = \cos^2(\theta_1) \cdot k_1^2$$
$$k_{n2}^2 = k_2^2 - k_{t2}^2 = k_2^2 - k_{t1}^2 = k_2^2 - \sin^2(\theta_1) \cdot k_1^2$$

Reflection/Transmission of TE-polarized field	Reflection/Transmission of TM-polarized field
$r_{12} = \frac{k_{n1} - k_{n2}}{k_{n1} + k_{n2}}$ $\quad r_{21} = \frac{k_{n2} - k_{n1}}{k_{n1} + k_{n2}}$	$r_{12} = \frac{n_1^2 k_{n2} - n_2^2 k_{n1}}{n_1^2 k_{n2} + n_2^2 k_{n1}}$ $\quad r_{21} = \frac{n_2^2 k_{n1} - n_1^2 k_{n2}}{n_1^2 k_{n2} + n_2^2 k_{n1}}$
$t_{12} = \frac{2k_{n1}}{k_{n1} + k_{n2}}$ $\quad t_{21} = \frac{2k_{n2}}{k_{n1} + k_{n2}}$	$t_{12} = \frac{2n_1 n_2 k_{n1}}{n_1^2 k_{n2} + n_2^2 k_{n1}}$ $\quad t_{21} = \frac{2n_1 n_2 k_{n2}}{n_1^2 k_{n2} + n_2^2 k_{n1}}$

Transfer matrices

Transmission through an interface

$$\mathbf{T}_{12} = \frac{1}{t_{12}} \begin{bmatrix} 1 & r_{12} \\ r_{12} & 1 \end{bmatrix}$$

Free-space

$$\mathbf{P} = \begin{bmatrix} e^{jk_n L} & 0 \\ 0 & e^{-jk_n L} \end{bmatrix}$$

Total matrix – Multiply **T** and **P** matrices

$$\mathbf{M} = \mathbf{M}_1 \mathbf{M}_2 \mathbf{M}_3 = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix}$$

Transmission and reflection

Field:

$$t = \frac{a_{n+N}}{a_n} = \frac{1}{M_{11}} \qquad r = \frac{b_n}{a_n} = \frac{M_{21}}{M_{11}}$$

Power:

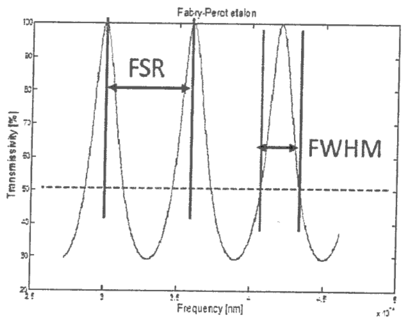
$$T = \frac{n_2 \cos(\theta_2)}{n_1 \cos(\theta_1)} |t|^2 \qquad R = |r|^2 \qquad T + R = 1$$

Electric field from the matrix elements

$$E_1 = a_1 e^{-jk_{y1}\cdot y} + b_1 e^{jk_{y1}\cdot y}$$
$$E_2 = a_2 e^{-jk_{y2}\cdot y} + b_2 e^{jk_{y2}\cdot y}$$

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix}$$

Fabry-Perot etalon (or F-P interferometer)



$$T = tt^* = \frac{(1-R)^2}{(1-R)^2 + 4R\cdot \sin^2\left(\frac{\omega}{2FSR}\right)}$$

Positions of the transmission peaks

$$\nu_N = N\frac{c_0}{2n_2L}[Hz]$$

$$\lambda_N = \frac{2n_2L}{N}[m]$$

Free spectral range

$$FSR = \frac{c_0}{2n_2L}[Hz] = \frac{\lambda_0^2}{2n_2L}[m]$$

For dispersive materials n_2 is group index

Linewidth (Full Width at Half Maximum)

$$\Delta\nu_{FWHM} = \frac{2FSR}{\pi} a\sin\left[\frac{(1-R)}{2\sqrt{R}}\right] \approx \frac{FSR}{\pi}\frac{(1-R)}{\sqrt{R}}$$

Quality factor (Q-factor)

$$Q = \frac{\omega_0}{\Delta\omega} \approx \frac{\lambda_0}{\Delta\lambda}$$

Decay of the field in the cavity

$$E(t) = E_0 e^{-\frac{\omega_0}{Q}t} = E_0 e^{-\frac{t}{t_c}}$$

t_c is photon lifetime

Waveguides

Fields of the modes

$$\mathbf{E}(x,y,z) = \mathbf{E}(x,y)e^{-j\beta z}$$
$$\mathbf{H}(x,y,z) = \mathbf{H}(x,y)e^{-j\beta z}$$

β is propagation constant of the mode

Effective index

$$n_{eff} = \frac{\beta}{\omega / c_0}$$

Group velocity

$$v_g = 1/\beta'(\omega_0) = \frac{d\omega}{d\beta} = \frac{c_0}{n_g}$$

Group index

$$n_g = n_{eff}(\omega) + \omega\frac{dn_{eff}}{d\omega} = n_{eff}(\lambda) - \lambda\frac{dn_{eff}}{d\lambda}$$

Transfer function

$$H(\omega) = A(\omega)e^{-j\phi(\omega)}$$

Group delay

$$\tau_g = -\frac{d\phi(\omega)}{d\omega}$$

Group delay dispersion

$$\frac{d\tau_g}{d\omega} = \frac{d\lambda}{d\omega}\frac{d\tau_g}{d\lambda}$$

Periodic structures

Dispersion relation of a periodic structure

$$\cos(K\Lambda) = \cos(k_1a)\cos(k_2b) - \frac{1}{2}\left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right)\sin(k_1a)\sin(k_2b)$$

This gives the relation between the Bloch wavenumber, K, and ω (for TE-polarization)

Phase matching:

$$\Delta\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$$

Diffraction equation (transmission grating):

$$n_1 \sin\theta_i - n_2 \sin\theta_d = \frac{\lambda_0}{\Lambda}m$$

Coupled-mode equations :

$$\frac{\partial}{\partial z}A_1(z) = -j\frac{|\beta_1|}{\beta_1}A_2(z)D_{21}^m e^{j(\beta_1-\beta_2-m\frac{2\pi}{\Lambda})z}$$

$$\frac{\partial}{\partial z}A_2(z) = -j\frac{|\beta_2|}{\beta_2}A_1(z)D_{12}^{-m} e^{j(\beta_2-\beta_1+m\frac{2\pi}{\Lambda})z}$$

Coupling coefficient

$$\kappa = D_{nk}^m = \left[D_{nk}^{-m}\right]^* = \frac{\omega}{4}\int_{xy}\mathbf{E}_k \cdot \varepsilon_m(x,y)\mathbf{E}_n d\mathbf{S}$$

Coupling coefficients for gratings:

Rectangular grating

$$\kappa_m \approx 2\frac{\Delta n}{m\lambda}$$

Sinusoidal grating

$$\kappa = \frac{\pi}{\lambda}n_1$$

Solution of the coupled-mode equation for counter-propagating modes (Bragg gratings) as a T-matrix

$$\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \overbrace{\begin{bmatrix} \cosh(bL) + j\frac{\Delta\beta}{2b}\sinh(bL) & j\frac{\kappa}{b}\sinh(bL) \\ -j\frac{\kappa^*}{b}\sinh(bL) & \cosh(bL) - j\frac{\Delta\beta}{2b}\sinh(bL) \end{bmatrix}}^{M} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$a_0(z) = A(z)e^{-j\beta z} \qquad b_0(z) = B(z)e^{j\beta z}$$

$$b = \sqrt{|\kappa|^2 - \left(\frac{\Delta\beta}{2}\right)^2} \qquad \Delta\beta = 2\beta - m\frac{2\pi}{\Lambda} \qquad t = \frac{a_{n+N}}{a_n} = \frac{1}{M_{11}} \qquad r = \frac{b_n}{a_n} = \frac{M_{21}}{M_{11}}$$

Coupled waveguides

Solution for co-propagating modes (Directional couplers)

$$\begin{bmatrix} A_1(z)e^{-j(\Delta\beta/2)z} \\ A_2(z)e^{j(\Delta\beta/2)z} \end{bmatrix} = \begin{bmatrix} \cos(bz) - j\frac{\Delta\beta}{2b}\sin(bz) & -j\frac{\kappa}{b}\sin(bz) \\ -j\frac{\kappa^*}{b}\sin(bz) & \cos(bz) + j\frac{\Delta\beta}{2b}\sin(bz) \end{bmatrix} \begin{bmatrix} A_1(0) \\ A_2(0) \end{bmatrix}$$

$$b = \sqrt{|\kappa|^2 + \left(\frac{\Delta\beta}{2}\right)^2} \qquad \Delta\beta = \beta_1 - \beta_2$$

General coupler (similar waveguides)

$$\begin{bmatrix} A_1(L) \\ A_2(L) \end{bmatrix} = \begin{bmatrix} \cos(\kappa L) & -j\sin(\kappa L) \\ -j\sin(\kappa L) & \cos(\kappa L) \end{bmatrix} \begin{bmatrix} A_1(0) \\ A_2(0) \end{bmatrix}$$

50/50 - Coupler

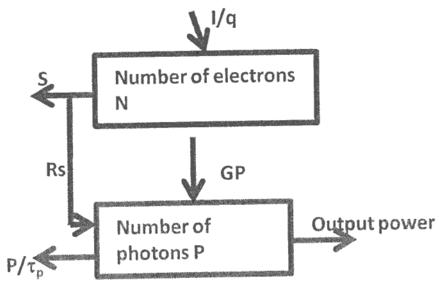
$$\kappa L = \frac{\pi}{4}m, \ m = 1,3,5,...$$

Lasers

Threshold condition for a Fabry-Perot laser

$$G_{th} = \alpha - \frac{1}{2L}\ln(R_1 R_2)$$

$$G_{th} = \alpha - \frac{1}{L}\ln(R), R_1 = R_2 = R$$



Rate equations to model the lasing dynamics

$$\frac{dN}{dt} = \frac{I}{q} - GP - \frac{N}{\tau_c}$$

$$\frac{dP}{dt} = GP - \frac{P}{\tau_p} + R_s$$

$$\tau_p = \frac{1}{\alpha_{cavity} v_g}$$

$$P_{Out} = \frac{1}{2}(\alpha_{mirror} v_g) h\nu P = \frac{1}{2}\left(\frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)\frac{c_0}{n_g}\right) h\nu P$$

Detectors

Photon energy $E = h\nu$

Responsivity $R = \eta\frac{e}{h\nu} = \eta\frac{e}{hc}\lambda$ Photocurrent $I = P_{in}R$

Signal-to-noise ratio

$$SNR = \frac{I^2}{\left(I_{shot}^2 + I_{thermal}^2 + I_{RIN}^2\right)} = \frac{I^2}{\left(2eI + \frac{4kT}{R} + RIN \cdot I^2\right)B}$$

Constants

c_0=299 792 458 m·s^{-1}
e=1.602 × 10^{-19} C
k= 1.380 × 10^{-23} J·K^{-1}
h=6.626 × 10^{-34} J·s

Power

$$P[dBm] = 10\log\left(\frac{P[W]}{1mW}\right)$$

$$0dBm = 1mW$$

Trigonometry

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$
$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\beta)\sin(\alpha)$$
$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$
$$\cos(2\alpha) = 1 - 2\sin^2(\alpha)$$